5) x^4+2x^3-1

$f'(x)=4x^3+6x^2=2x^2(2x+3)$

crit pts

$f'=0$

$2x^2(2x+3)=0$

$\Rightarrow x=0, x=-\frac{3}{2}$

[stationary points]

f' undefined nowhere

sign of f': $- - - \mid + + + \mid + + +$

$-10 \quad -\frac{3}{2} \quad -1 \quad 0 \quad 5$

inc: $[-\frac{3}{2}, \infty)$

dec: $(-\infty, -\frac{3}{2}]$

rel min: @ $x=-\frac{3}{2}$

$f''(x)=12x^2+12x=12x(x+1)$

Possible inflection Points

$f''=0$

$12x(x+1)=0$

$\Rightarrow x=0, -1$

f'' undefined nowhere

sign of f'': $+ + + \mid - - - \mid + + +$

$-10 \quad -1 \quad -\frac{1}{2} \quad 0 \quad 10$

c-up: $(-\infty,-1)\cup(0,\infty)$

c-dn: $(-1,0)$

pts of inflection: @ $x=-1, x=0$

y-int: $f(0) = (0^4) + 2(0)^3 - 1 = -1$

x-int: IDK

Symmetry

$$f(-x) = (-x)^4 + 2(-x)^3 - 1$$

$$= x^4 - 2x^3 - 1$$

cpare with $f(x)$

$f(x) = f(-x)$

f^n is even

(symm. about y-axis)

$-f(x) = f(-x)$

f^n is odd

(symm. about origin)

rotate 180°

7) $3x^5-5x^3$

$f'(x)=15x^4-15x^2=15x^2(x^2-1)$

$=15x^2(x-1)(x+1)$

crit pts

$f'=0$

$15x^2(x-1)(x+1)=0$

or $x=0,+1,-1$

f' und.

nowhere

sign of f'

+ + + | - - - | - - - | + + + + +

$-10 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad +1 \quad +10$

inc: $(-\infty,-1]\cup[1,\infty)$

dec: $[-1,1]$

rel min: @ 1

rel max: @ −1

$f'(x)=15x^4-15x^2$

$f''(x)=60x^3-30x$

$=30x(2x^2-1)$

possible pts of inflection

$f''=0$

$30x(2x^2-1)=0$

$x=0,-\sqrt{\frac{1}{2}},\sqrt{\frac{1}{2}}$

f'' und

nowhere

− − − − | + + + + | − − − − | + + + +

$-10 \quad -\frac{\sqrt{2}}{2} \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad \frac{\sqrt{2}}{2}\approx .707 \quad 10$

c-up: $\left(-\frac{\sqrt{2}}{2},0\right)\cup\left(\frac{\sqrt{2}}{2},\infty\right)$

c-dn: $\left(-\infty,-\frac{\sqrt{2}}{2}\right)\cup\left(0,\frac{\sqrt{2}}{2}\right)$

pts of inflection

At: $x=-\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2}$

symmetries

$f(x)=3x^5-5x^3$

$f(-x)=3(-x)^5-5(-x)^3$

$=-3x^5+5x^3$

$=f(x)$? No NOT EVEN

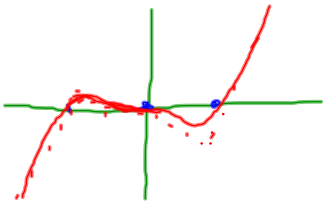
$=-f(x)$? Yes! odd

y-int: $f(0) = 3(0)^5 - 5(0)^3 = 0$

x-int: $3x^5 - 5x^3 = 0$

$x^3(3x^2 - 5) = 0$

$\Rightarrow$ roots @ $x = 0$ (multiplicity 3)



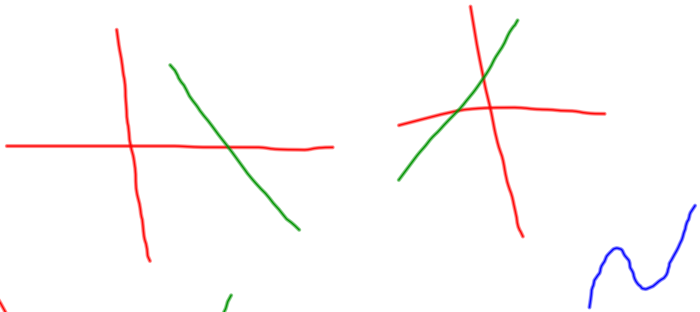
$x = -\sqrt{\frac{5}{3}}$ (mult 1)

$x = \sqrt{\frac{5}{3}}$ (mult 1)

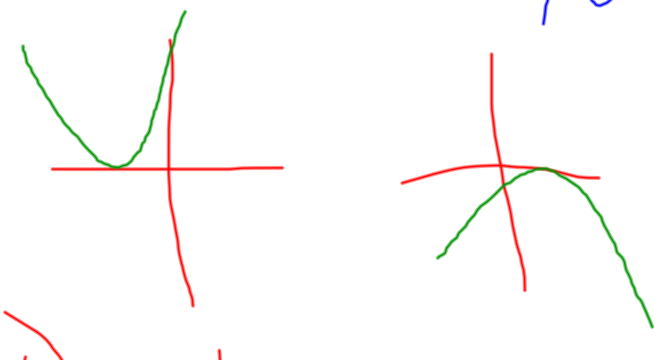
$f(x) = 3x^3\left(x + \sqrt{\frac{5}{3}}\right)^1\left(x - \sqrt{\frac{5}{3}}\right)^1$

multiplicities

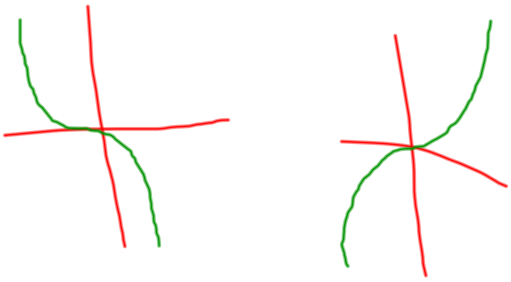
1)



even)



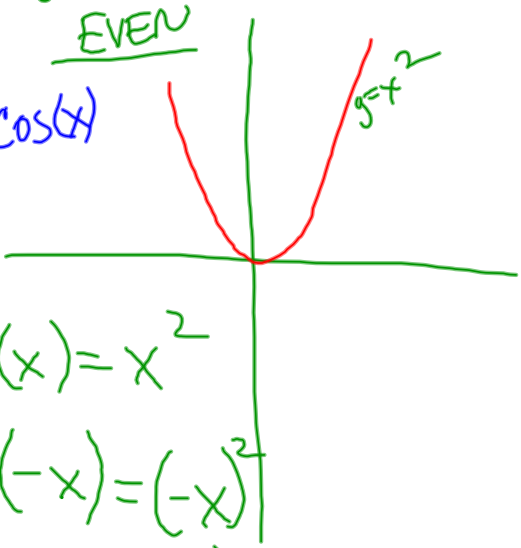
odd>1)



Symmetries

EVEN

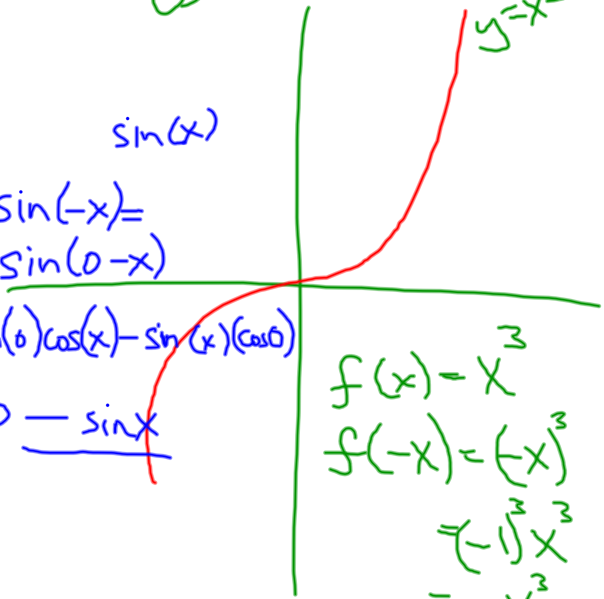
$\cos(x)$



$$f(x) = x^2$$

$$f(-x) = (-x)^2$$
$$= x^2$$

ODD



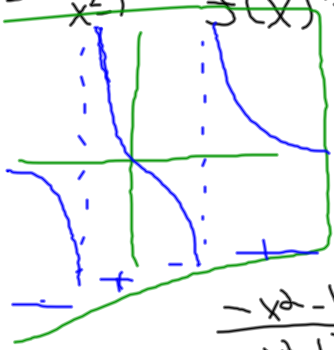
$\sin(x)$

$\sin(-x) =$
$\sin(0 - x)$
$= \sin(0)\cos(x) - \sin(x)(\cos 0)$
$= 0 - \sin x$

$$f(x) = x^3$$

$$f(-x) = (-x)^3$$
$$= (-1)^3 x^3$$
$$= -x^3$$

12) $\frac{x}{x^2-1}$



$f'(x) = \frac{(x)'(x^2-1) - (x)(x^2-1)'}{(x^2-1)^2}$

$\frac{(x^2-1) - (x)(2x)}{(x^2-1)^2}$ $(x^2-1) - 2x^2$

$f(-x) = \frac{(-x)}{(-x)^2-1}$

$= \frac{-x}{x^2-1}$

$\frac{-x^2-1}{(x^2-1)^2}$

$f'(x)=0$ $\quad$ $f'(x)$ undefine

$(x^2-1)^2=0$

$x^2-1=0$

$x^2=1$

$x=\pm1$

$-x^2-1=0$

$-x^2=1$

$x^2=-1$

$x=$ doesn't exist

$- - - | - - - | - - -$
$-10 \quad -1 \quad 0 \quad 1 \quad 10$

~~same~~ not defined at $x=-1,1$

$\cancel{\text{inc}}$: $(-\infty,-1)$
$(1,\infty)$
$(-1,1)$

$\frac{-x^2-1}{(x^2-1)^2}$ $\quad (-x^2-1)(x^2-1)^2$

$\frac{(-2x)(x^2-1)^2 - 2(x^2-1)(2x)(-x^2-1)}{[(x^2-1)^2]^2}$

$\frac{[(-2x)(x^2-1)][(x^2-1)+2(-x^2-1)]}{(x^2-1)^4}$

$f''(x)=0$ $\quad$ $f''(x)$ undefine

$[(-2x)(x^2-1)][(x^2-1)+2(-x^2-1)]=0$

$(-2x)(x^2-1)(-x^2-3)$

$x=0$ $\quad$ too $\quad$ ~~app~~ for

$(x^2-1)^4=0$

$x^2-1=0$

$x^2=1$

$x=\pm1$

$(x^2-1)+-2x^2-2=0$

$-x^2-3=0$

$-x^2=3$

$x^2=-3$

$x\neq$ exist

$(-)(-)(-)$

$- - - | + + + | - - - | + + +$
$-10 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1 \quad 10$

C: up
$(-1,0)$
$(1,\infty)$

C: down
$(-\infty,-1)$
$(0,1)$

pts of inflection occur when $x=(-1,0,1)$

y-int: $f(0) = \frac{0}{0^2-1} = 0$

x-int: $x = 0 \Rightarrow x = 0$

v-a: $x = +1, -1$

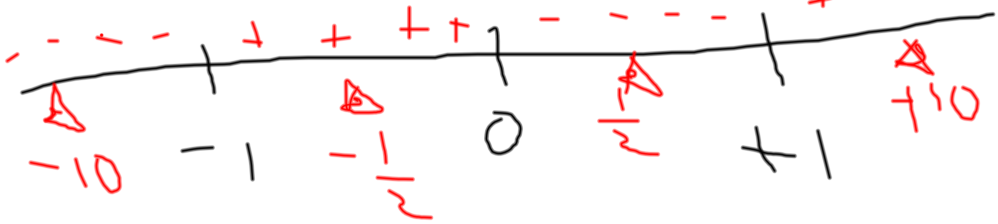
h-a: $\lim\limits_{x \to \infty} \frac{x}{x^2-1} = 0$

$$\lim_{x \to -\infty} \frac{x}{x^2-1} = 0$$

Sign chart of $f(x)$

sign chart of f(x)

on # line, label roots & vertical asymp



#29 D&I madness

$$\int \frac{1}{2x^3}\,dx$$

$$= \frac{1}{2}\int \frac{1}{x^3}\,dx = \frac{1}{2}\int x^{-3}\,dx$$

$$= \frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + C = -\frac{1}{4x^2} + C$$

23)

$$f(x)=\frac{(x-1)^2}{x^2} = \frac{x^2-2x+1}{x^2} = 1-\frac{2}{x}+\frac{1}{x^2}$$

$$f' = \frac{2}{x^2}-\frac{2}{x^3}=0$$

$$\frac{2x-2}{x^3} \Rightarrow x=1 \ (=0), \quad x=0 \ (\text{und})$$

$$f'(x)=\frac{(x-1)^{2\prime}(x^2)-(x^2)'(x-1)^2}{(x^2)^2}$$

$$\frac{2(x-1)(x^2)-(2x)(x-1)^2}{(x^4)} = \frac{2x(x-1)[x-(x-1)]}{x^4} = \frac{2x(x-1)(1)}{x^4} \quad \frac{2x^2-2x}{x^4}$$

0@ undefined@ 0

1

+ + + + , - - - - , + + + + +

-10 0 1/2 1 10

$$\frac{2(-10-1)(10^2)-(-20)(-11)^2}{(-10)^4} \ +$$

$$2(9)(100)-(20)(81) \ +$$

$$2(-\tfrac{1}{2})(\tfrac{1}{4})-(1)(-1) \ +$$

increasing @ $-\infty \to 0$, $1 \to +\infty$

Decreasing @ $0 \to 1$

$$f''(x) \ \frac{2x(x-1)(1)}{x^4}$$

$$f'' : \frac{(4x-2)x^4-(2x^2-2x)(4x^3)}{x}$$

$$(2x(x-1))' = (2x)'(x-1)+(x-1)'(2x)$$

$2x-2+2x$

$$\frac{4x-2}{4x^3} = f''(x) \to 2(2x-1)$$

0@ 1/2 undefined@

- - - - - - - - - + + + +

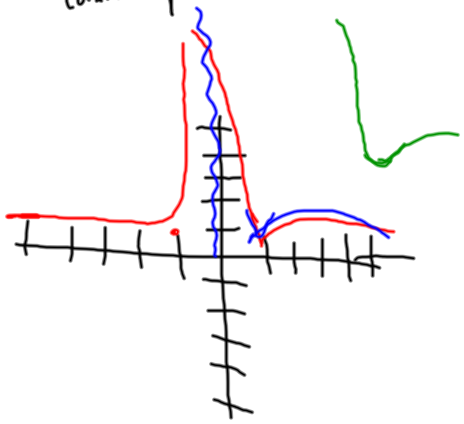
-2 0 1/4 1/2 2

concave down $-\infty \to 1/2$ $(-\infty,0) \cup (0,\tfrac{1}{2})$

concave up $1/2 \to 2$

$$f(x)=\frac{(x-1)^2}{x^2} \Rightarrow \frac{x^2-2x+1}{x^2} \quad (x-1)(x-1)$$

$$\lim_{x\to+\infty} = 1 \quad +00$$



$$\lim_{x\to-\infty} \left(\frac{x^2}{x^2}\right) - \frac{2}{x} + \frac{1}{x^2}$$

$1-0+0$

$$\lim_{x\to\infty} = 1$$

HA: $\lim_{x \to \infty} f(x)$ $\lim_{x \to -\infty} f(x)$

VA: $\lim_{x \to a} f(x) = +\infty?$ $-\infty?$

$\frac{(x-1)(x-2)}{(x-1)} = x-2, \; x \neq 1$

Vertical Asymptotes occur when denominator $= 0$

[precalc: determine after f(x) is in "reduced" form]

5.3 homework 2010-12-10 Pd 3